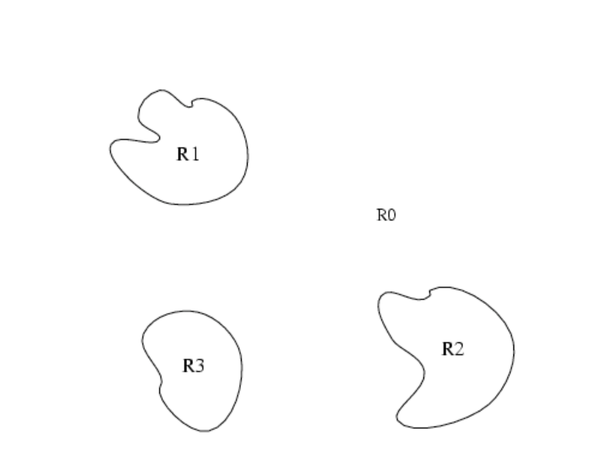
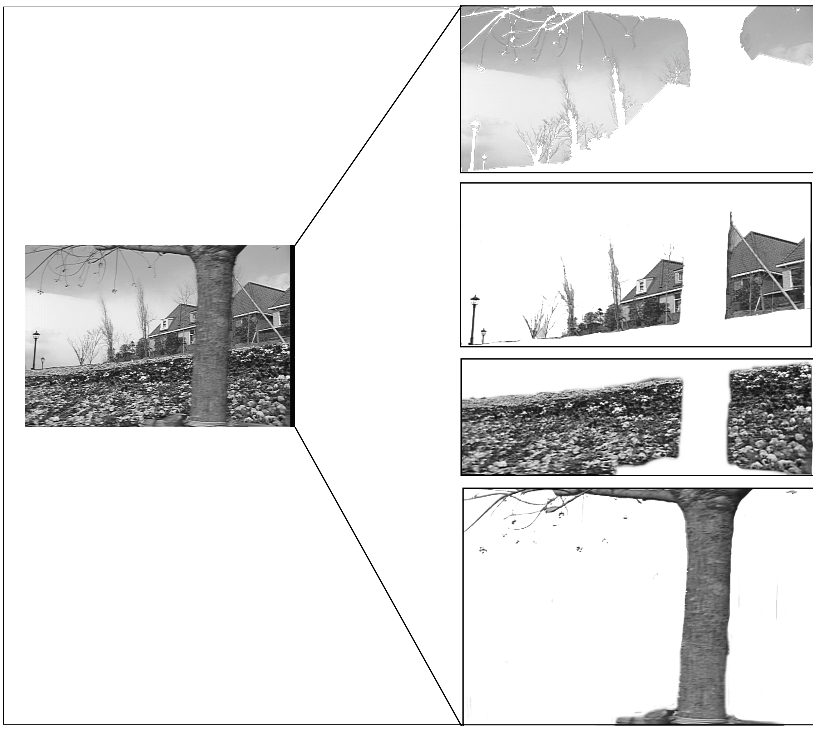
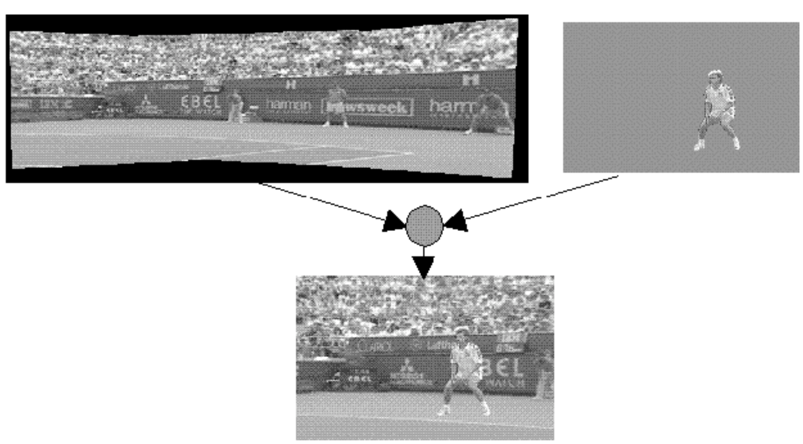
Region Segmentation

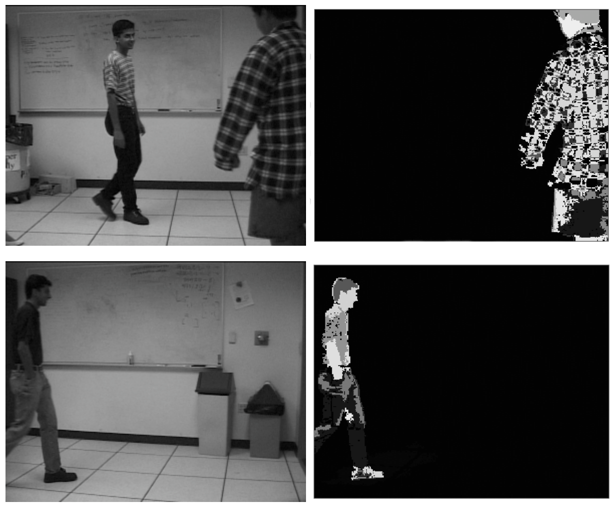
Segmentation







Initial Segmentation



Segmentation

- Partition $f(x,y)$ into sub-images: $R_1, R_2, \ldots, R_n$ such that the following constraints are satisfied:
  - $\bigcup_{i=1}^{n} R_i = f(x,y)$
  - $R_i \cap R_j = \emptyset, i \neq j$
  - Each sub-mage satisfies a predicate or set of predicates
    - All pixels in any sub-image musts have the same gray levels.
    - All pixels in any sub-image must not differ more than some threshold
    - All pixels in any sub-image may not differ more than some threshold from the mean of the gray of the region
    - The standard deviation of gray levels in any sub-image must be small.

Simple Segmentation

$$B(x,y) = \begin{cases} 1 & \text{if } f(x,y) < T \\ 0 & \text{Otherwise} \end{cases}$$

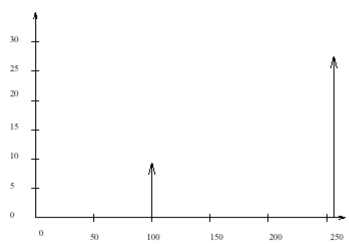
$$B(x,y) = \begin{cases} 1 & \text{if } T_1 < f(x,y) < T_2 \\ 0 & \text{Otherwise} \end{cases}$$

$$B(x,y) = \begin{cases} 1 & \text{if } f(x,y) \in Z \\ 0 & \text{Otherwise} \end{cases}$$

Histogram

Histogram graphs the number of pixels in an image with a Particular gray level as a function of the image of gray levels.

255	255	255	255	255	255
255	255	255	255	255	255
255	255	100	100	100	255
255	255	100	100	100	255
255	255	100	100	100	255
255	255	255	255	255	255

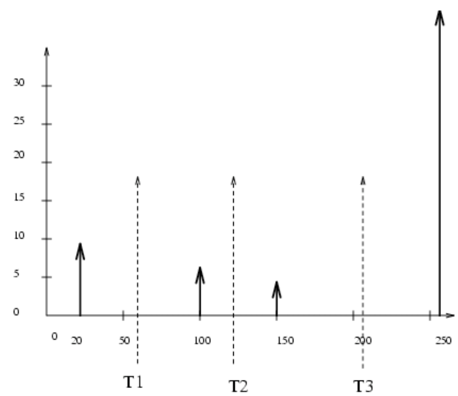



```
For (I=0, I<m, I++)
 For (J=0, J<m, J++)
   histogram[f(I,J)]++;
```

imhist

Example

255	255	255	255	255	255	255	20
255	255	255	100	100	255	20	20
255	255	255	100	100	255	20	20
255	255	255	100	100	255	20	20
255	255	255	255	255	255	20	20
255	255	255	255	255	255	255	255
150	150	255	255	255	255	255	255
150	150	255	255	255	255	255	255



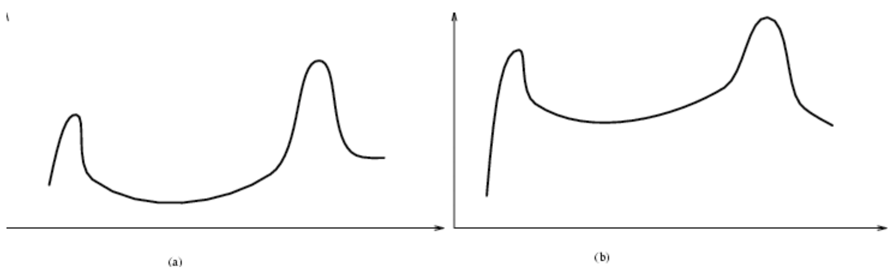
Segmentation Using Histogram

$$B_1(x,y) = \begin{cases} 1 & \text{if } 0 < f(x,y) < T_1 \\ 0 & \text{Otherwise} \end{cases}$$

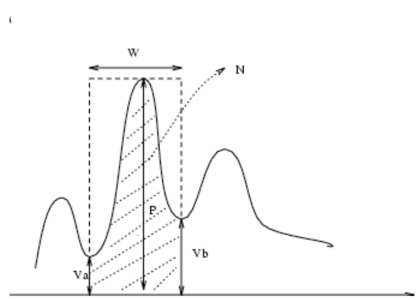
$$B_2(x,y) = \begin{cases} 1 & \text{if } T_1 < f(x,y) < T_2 \\ 0 & \text{Otherwise} \end{cases}$$

$$B_3(x,y) = \begin{cases} 1 & \text{if } T_2 < f(x,y) < T_3 \\ 0 & \text{Otherwise} \end{cases}$$

Realistic Histogram



Peakiness Test



$$Peakiness = \left(1 - \frac{(V_a + V_b)}{2P}\right) \cdot \left(1 - \frac{N}{(W.P)}\right)$$

Connected Component

$$
\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}
\quad
\begin{bmatrix} 0 & 0 & 0 & a & 0 \\ b & b & 0 & a & a \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c & c & 0 \\ 0 & d & 0 & c & 0 \end{bmatrix}
$$

4

Connectedness

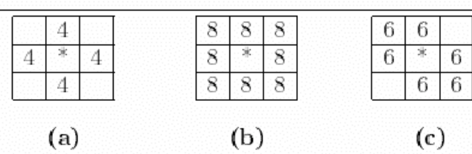


Figure 3.6: Pixel connectedness. (a) 4-connected. (b) 8-connected. (c) 6-connected.

Connected Component

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 & a & 0 \\ b & b & 0 & a & a \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c & c & 0 \\ 0 & c & 0 & c & 0 \end{bmatrix}$$

8

Recursive Connected Component Algorithm

1. Scan the binary image left to right, top to bottom.
2. If there is an unlabeled pixel with a value of '1' assign a new label to it.
3. Recursively check the neighbors of the pixel in step 2 and assign the same label if they are unlabeled with a value of '1'.
4. Stop when all the pixels of value '1' have been labeled.

Figure 3.7: Recursive Connected Component Algorithm.

Sequential

$$
\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}
\qquad
\begin{bmatrix} 0 & 0 & 0 & a & 0 \\ b & b & 0 & a & a \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c & c & 0 \\ 0 & d & c & c & 0 \end{bmatrix}
\quad d=c
$$

Sequential Connected Component Algorithm

1. Scan the binary image left to right, top to bottom.
2. If an unlabeled pixel has a value of '1', assign a new label to it according to the following rules:

$$
\begin{array}{cc} & 0 \\ 0 & 1 \end{array} \rightarrow \begin{array}{cc} & 0 \\ 0 & L \end{array}
\qquad\qquad
\begin{array}{cc} & 0 \\ L & 1 \end{array} \rightarrow \begin{array}{cc} & 0 \\ L & L \end{array}
$$

$$
\begin{array}{cc} & L \\ 0 & 1 \end{array} \rightarrow \begin{array}{cc} & L \\ 0 & L \end{array}
\qquad\qquad
\begin{array}{cc} & L \\ M & 1 \end{array} \rightarrow \begin{array}{cc} & L \\ M & L \end{array} \text{ (Set } L = M\text{).}
$$

3. Determine equivalence classes of labels.
4. In the second pass, assign the same label to all elements in an equivalence class.

Figure 3.8: Sequential Connected Component Algorithm.

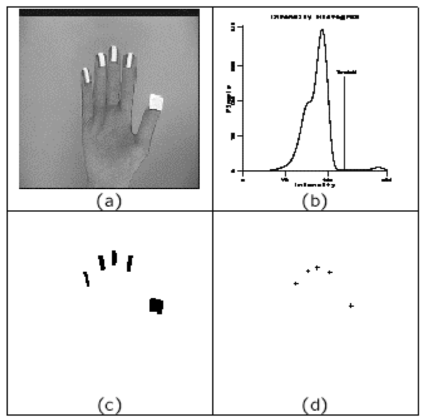
Recursive

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 & a & 0 \\ b & b & 0 & a & a \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c & c & 0 \\ 0 & c & c & c & 0 \end{bmatrix}$$

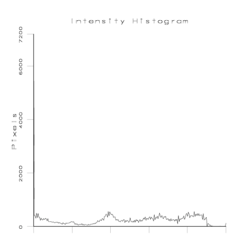
Steps in Segmentation Using Histogram

1. Compute the histogram of a given image.
2. Smooth the histogram by averaging peaks and valleys in the histogram.
3. Detect good peaks by applying thresholds at the valleys.
4. Segment the image into several binary images using thresholds at the valleys.
5. Apply connected component algorithm to each binary image find connected regions.

Example: Detecting Fingertips

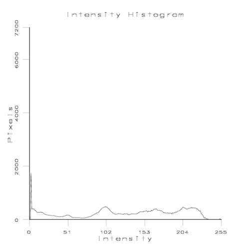


Example-II

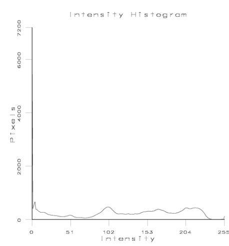


93 peaks

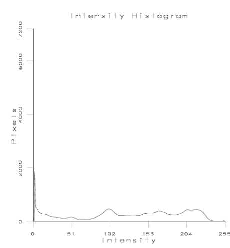
Smoothed Histograms



Smoothed histogram
(averaging using mask
Of size 5)
54 peaks (once)
After peakiness 18



Smoothed histogram
21 peaks (twice)
After peakiness 7



Smoothed histogram
11 peaks (three times)
After peakiness 4

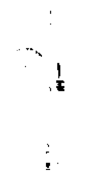
Regions

(0,40)

(40, 116)

Regions

(116,243)



(243,255)

Geometrical Properties

Area

$$A = \sum_{x=0}^{m} \sum_{y=0}^{n} B(x, y)$$

Centroid

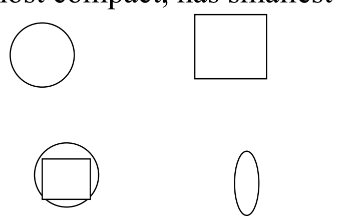
$$\bar{x} = \frac{\sum_{x=0}^{m} \sum_{y=0}^{n} xB(x, y)}{A}, \quad \bar{y} = \frac{\sum_{x=0}^{m} \sum_{y=0}^{n} yB(x, y)}{A}$$

Perimeter & Compactness

Perimeter: The sum of its border points of the region. A pixel which has at least one pixel in its neighborhood from the background is called a border pixel.

Compactness $C = \frac{P^2}{4\pi A}$

Circle is the most compact, has smallest value



Area decreases

Orientation of the Region

Least second moment

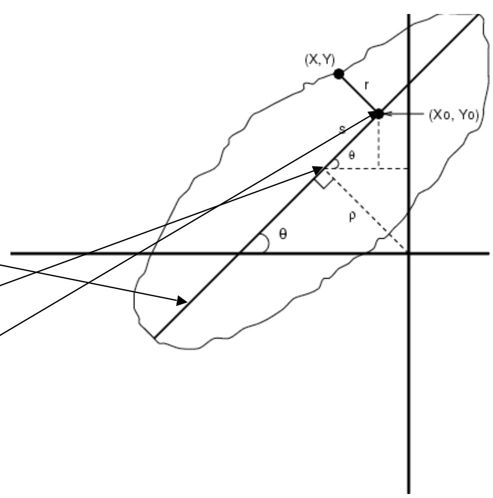
Minimize

$$E = \iint r^2 B(x, y)\,dx\,dy$$

$$x \sin\theta - y \cos\theta + \rho = 0$$

$$(-\rho \sin\theta, \rho \cos\theta)$$

$$x_0 = -\rho \sin\theta + s \cos\theta$$
$$y_0 = \rho \cos\theta + s \sin\theta$$



Orientation of the Region

$$r^2 = (x\sin\theta - y\cos\theta + \rho)^2$$

$$E = \iint r^2 B(x,y)\,dx\,dy$$

$$E = \iint (x\sin\theta - y\cos\theta + \rho)^2 B(x,y)\,dx\,dy$$

Substitute r in E and differentiate Wrt ρ and equate it to zero

$$A(\bar{x}\sin\theta - \bar{y}\cos\theta + \rho) = 0$$

$(\bar{x},\bar{y})$ is the centroid

$$x' = x - \bar{x},\quad y' = y - \bar{y}$$

$$E = a\sin^2\theta - b\sin\theta\cos\theta + c\cos^2\theta$$

$$E = \frac{1}{2}(a+c) - \frac{1}{2}(a-c)\cos 2\theta - \frac{1}{2}b\sin 2\theta$$

Substitute value of ρ

$$a = \iint (x')^2 B(x,y)\,dx'\,dy'$$

$$b = \iint x'y' B(x,y)\,dx'\,dy'$$

$$c = \iint (y')^2 B(x,y)\,dx'\,dy'$$

Orientation of the Region

$$E = \frac{1}{2}(a+c) - \frac{1}{2}(a-c)\cos 2\theta - \frac{1}{2}b\sin 2\theta$$

Differentiating this wrt θ

$$\tan 2\theta = \frac{b}{a-c}$$

$$\sin 2\theta = \pm\frac{b}{\sqrt{b^2+(a-c)^2}}$$

$$\cos 2\theta = \pm\frac{a-c}{\sqrt{b^2+(a-c)^2}}$$

$$a = \iint (x')^2 B(x,y)\,dx'\,dy'$$

$$b = \iint x'y' B(x,y)\,dx'\,dy'$$

$$c = \iint (y')^2 B(x,y)\,dx'\,dy'$$

$$x' = x - \bar{x},\quad y' = y - \bar{y}$$

$$a = \sum\sum x^2 B(x,y) - A\bar{x}^2$$

$$b = 2\sum\sum xyB(x,y) - A\overline{xy}$$

$$c = \sum\sum y^2 B(x,y) - A\bar{y}^2$$

Moments

General Moments

$$m_{pq} = \iint x^p y^q B(x, y)\,dxdy$$

Discrete

$$M_x^1 = \sum_{x=0}^{m} \sum_{y=0}^{n} xB(x, y),\ M_y^1 = \sum_{x=0}^{m} \sum_{y=0}^{n} yB(x, y)$$

$$M_x^2 = \sum_{x=0}^{m} \sum_{y=0}^{n} x^2 B(x, y),\ \ M_y^2 = \sum_{x=0}^{m} \sum_{y=0}^{n} y^2 B(x, y),\ \ M_{xy}^2 = \sum_{x=0}^{m} \sum_{y=0}^{n} xyB(x, y)$$

Half size, mirror
Rotated 2, rotated 45

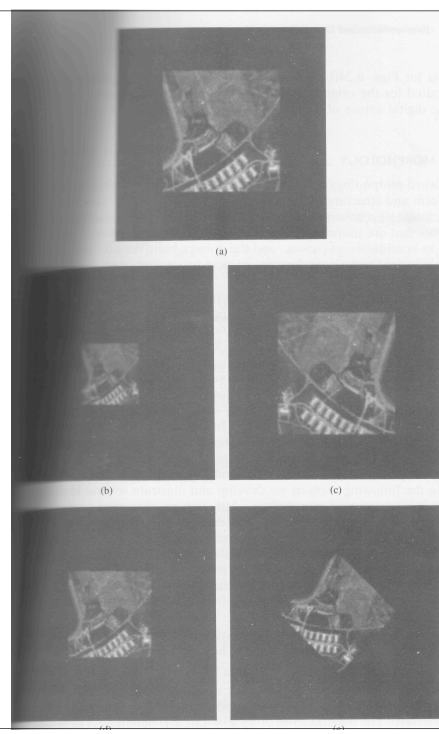


Table 8.2 Moment Invariants for the Images in Figs. 8.24(a)–(e)

Invariant (Log)	*Original*	*Half Size*	*Mirrored*	*Rotated 2°*	*Rotated [illegible]*
ϕ_1	6.249	6.226	6.919	6.253	6.318
ϕ_2	17.180	16.954	19.955	17.270	16.803
ϕ_3	22.655	23.531	26.689	22.836	19.724
ϕ_4	22.919	24.236	26.901	23.130	20.437
ϕ_5	45.749	48.349	53.724	46.136	40.525
ϕ_6	31.830	32.916	37.134	32.068	29.315
ϕ_7	45.589	48.343	53.590	46.017	40.470

Hu moments