Week 5

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Project Recap

Motivation

- Standard RL typically assumes a scalar reward
- Many problems have multiple objectives
- Trade-offs often need to be made

Question:

- How to effectively navigate objective trade-offs in a balanced way?

Existing Approach

Nash Bargaining Solution (Product Based)	Chebyshev / Generalized Maximin (Floor Based)
Product encourages joint improvement	Strict floor ensures worst objective prioritized
Smooth optimization for policy updates	Strong gradient signal even if objective drops to zero
No hard barrier [20,60]=1200 > [30,30]=900	Non-differentiable sharp corners
If one objective drops to zero, gradient unclear signal	Sensitive to scaling, requires tuning of reference points

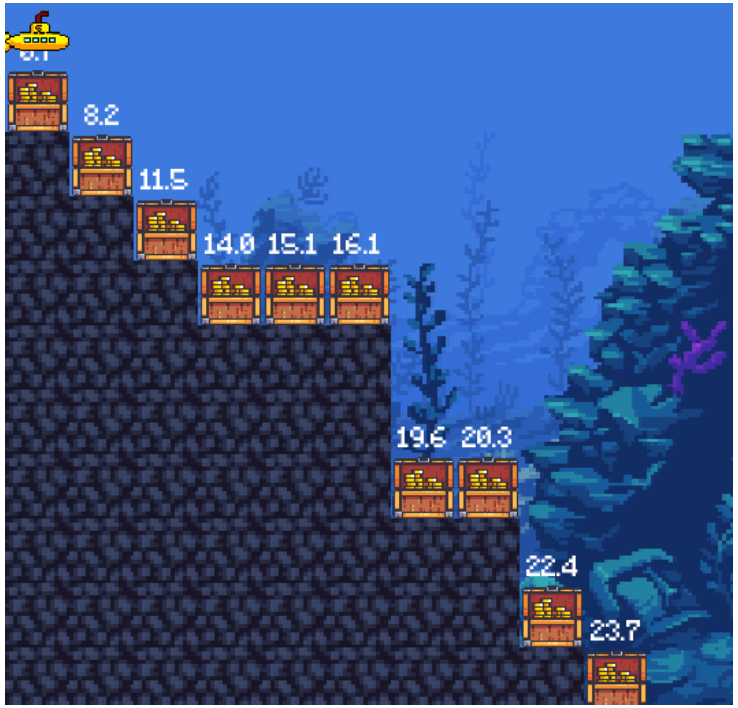
Our Method: Hybrid Nash + Chebyshev

$$f : \mathbb{R}^K \to \mathbb{R} \quad \boldsymbol{x} = [x_1, x_2, \ldots, x_K]^T \in \mathbb{R}^K$$

- Soft barrier with hard floor $f_{\text{hybrid}}(\boldsymbol{x}; \alpha) = \alpha f_{\text{nash}} + (1 - \alpha) f_{\text{Chebyshev}}$
- Clear gradient signal even when an objective goes to zero
- Smoothed optimization
- Reduced scale sensitivity and less need for tuning

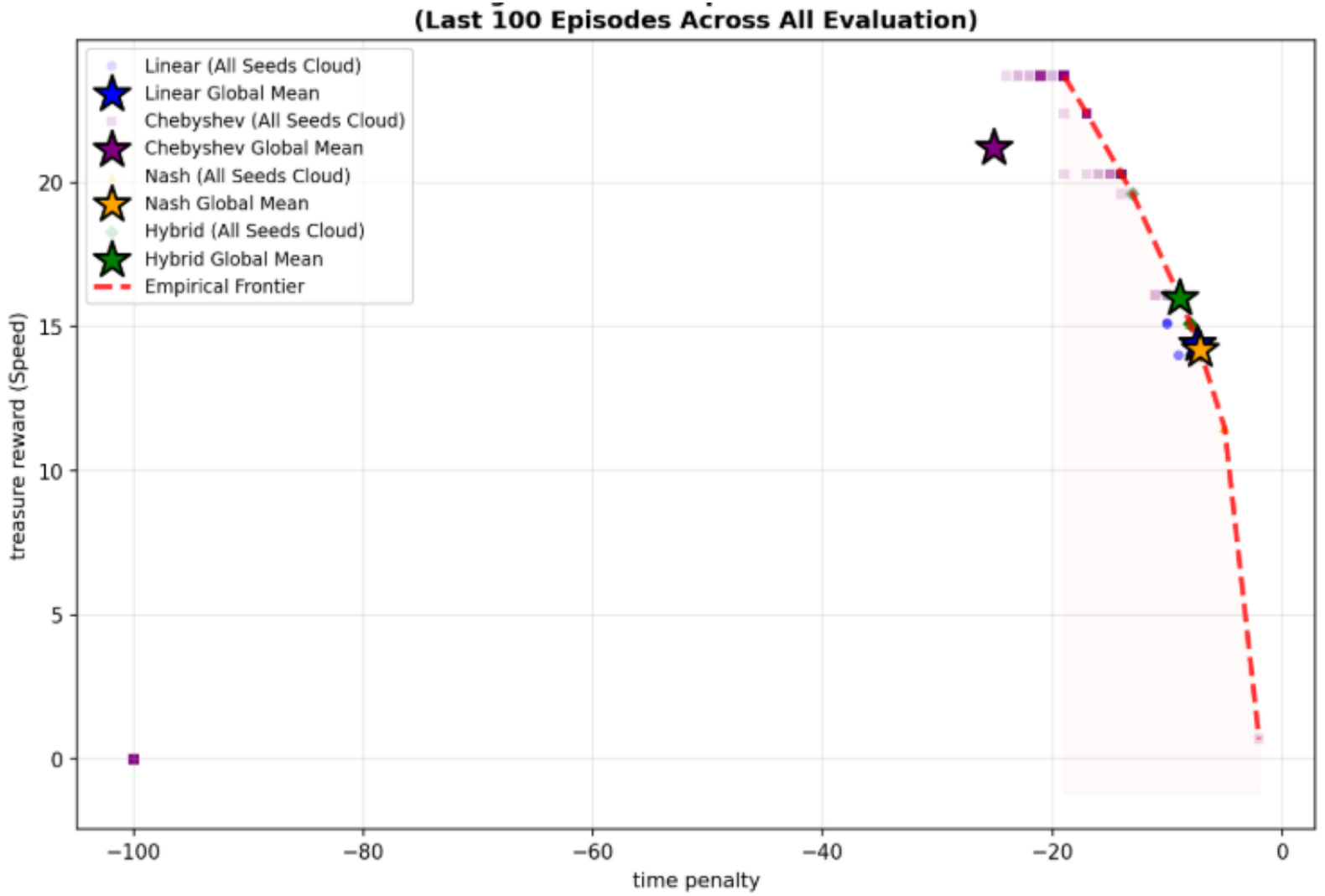
Environment

- deep-sea-treasure-v0
- 2 Objectives
  - Treasure Reward
  - Time Penalty
- Discrete action space
- Discrete state space



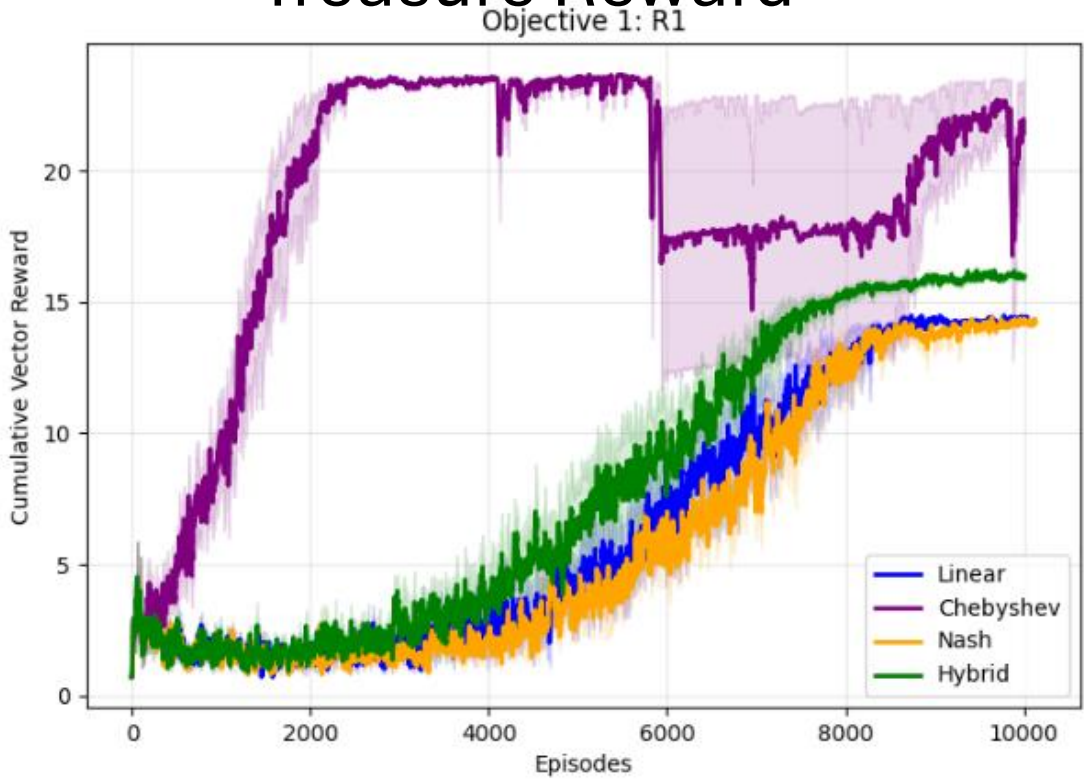
Results Tradeoffs

- Hybrid successfully navigated between Nash and Chebyshev

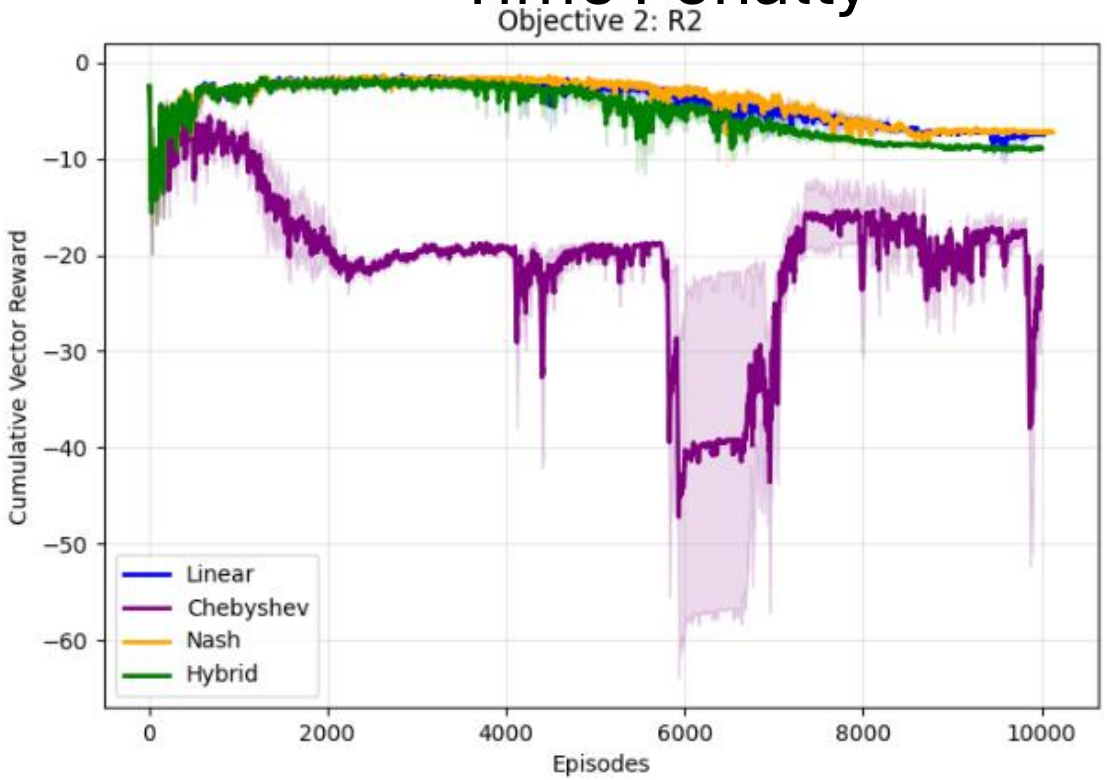


Results Continued

Treasure Reward

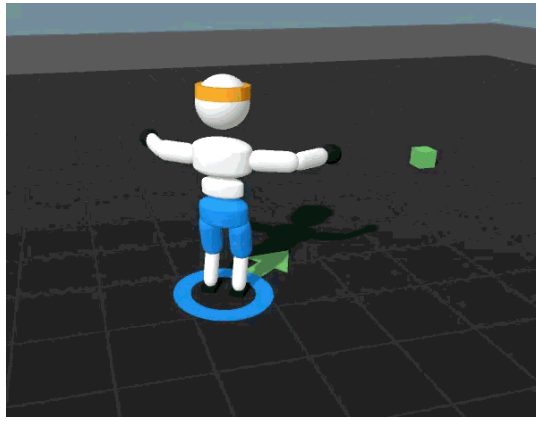


Time Penalty



Next Steps

- Evaluate on additional environments
- Hyperparameter Sweeps and Ablations
- Statistical Analysis and Paper Writing



Appendix

$$f : \mathbb{R}^K \to \mathbb{R} \quad \boldsymbol{x} = [x_1, x_2, \ldots, x_K]^T \in \mathbb{R}^K$$

- Nash Bargaining

$$f_{\text{Nash}}(\boldsymbol{x}) = \prod_{k=1}^{K} x_k.$$

- Chebyshev (Generalized Maximin)

$$\boldsymbol{z}^* = [z_1^*, z_2^*, \ldots, z_K^*]^T$$

$$f_{\text{Chebyshev}}(\boldsymbol{x}) = -\max_{k \in \{1,\ldots,K\}} [w_k (z_k^* - x_k)]$$

Hybrid Method

$$f_{\text{hybrid}}(\boldsymbol{x}; \alpha) = \alpha f_{\text{nash}} + (1 - \alpha) f_{\text{Chebyshev}}$$

Linear Scalarization

$$f_{\text{linear}}(\boldsymbol{x}) = \sum_{k=1}^{K} w_k x_k \qquad \sum_{k=1}^{K} w_k = 1$$

Thank You

- Questions?