Deep Accelerated Signal Processing (Week 8)

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Preconditioner - Results

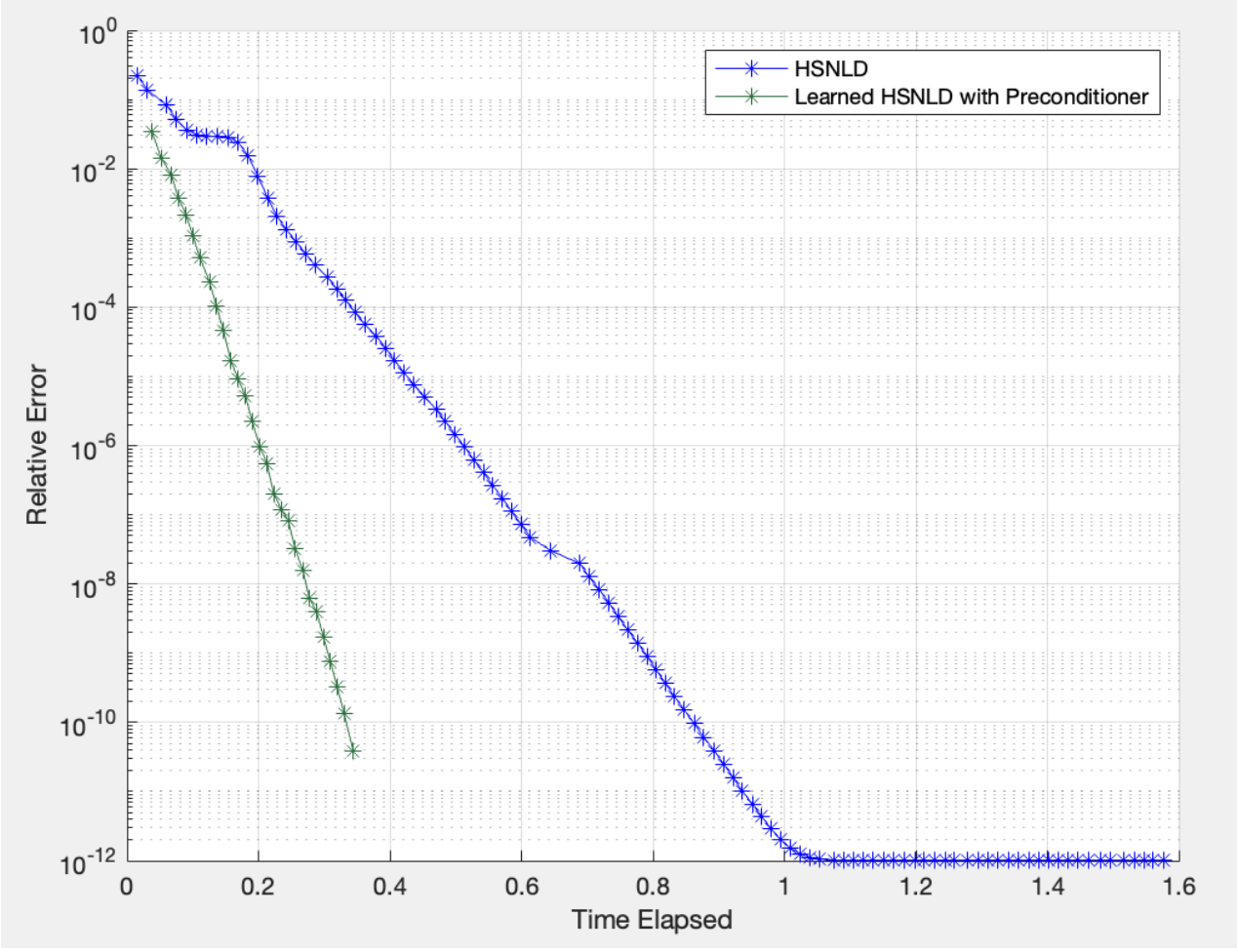


Figure: Matrix recovery with $n = 2^{13} - 1 = 8191$; $r = 20$; $\alpha = 0.2$; fully observed

Prior Algorithm

Algorithm 1 **H**ankel **S**tructured **N**ewton-**L**ike **D**escent (HSNLD)

Input: $\Pi_\Omega \boldsymbol{f}$: partial observation on the corrupted Hankel matrix in reweighted vector form; r: the rank of underlying Hankel matrix; p: observation rate; α: outlier density; $\{\gamma_k\}$: parameters for sparsification operator; η: step size; K: maximum number of iterations.

Initialization: Let $\boldsymbol{L}_0, \boldsymbol{R}_0$ be produced by (6).

for $k = 0, 1, \ldots, K-1$ **do**

$$\boldsymbol{z}_{k+1} = \mathcal{G}^*(\boldsymbol{L}_k \boldsymbol{R}_k^*)$$

$$\boldsymbol{s}_{k+1} = \Gamma_{\gamma_k \alpha p}(\Pi_\Omega(\boldsymbol{f} - \boldsymbol{z}_{k+1}))$$

$$\begin{bmatrix} \boldsymbol{L}_{k+1} \\ \boldsymbol{R}_{k+1} \end{bmatrix} = \mathcal{P}_{\mathcal{C}} \begin{bmatrix} \boldsymbol{L}_k - \eta \nabla_{\boldsymbol{L}} \ell(\boldsymbol{L}_k, \boldsymbol{R}_k; \boldsymbol{s}_{k+1}) \left(\boldsymbol{R}_k^* \boldsymbol{R}_k\right)^{-1} \\ \boldsymbol{R}_k - \eta \nabla_{\boldsymbol{R}} \ell(\boldsymbol{L}_k, \boldsymbol{R}_k; \boldsymbol{s}_{k+1}) \left(\boldsymbol{L}_k^* \boldsymbol{L}_k\right)^{-1} \end{bmatrix}$$

end for

Output: $\boldsymbol{z}_{\text{output}} = \mathcal{G}^*(\boldsymbol{L}_K \boldsymbol{R}_K^*)$: recovered Hankel matrix in vector form.

Figure: Original HSNLD Algorithm (Cai et al., 2025)

Proposed Algorithm

Input: $\Pi_\Omega \mathbf{f}$: Partially observed Hankel matrix in reweighted vector form; r: rank of the underlying Hankel matrix; p: observation rate; α: outlier density; K: number of layers; $\{\zeta_k\}$: learned thresholding values; $\{P_k\}$: learned matrix preconditioners

Proposed Algorithm

Input: $\Pi_\Omega \mathbf{f}$: Partially observed Hankel matrix in reweighted vector form; r: rank of the underlying Hankel matrix; p: observation rate; α: outlier density; K: number of layers; $\{\zeta_k\}$: learned thresholding values; $\{P_k\}$: learned matrix preconditioners

Initialization: Let $\mathbf{L}_0$, $\mathbf{R}_0$ be produced by spectral initialization

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for $k = 0, 1, \ldots, K-1$ do

$$\mathbf{z}_{k+1} = \mathcal{G}^*(\mathbf{L}_k \mathbf{R}_k^*)$$

$$\mathbf{s}_{k+1} = \mathcal{S}_{\zeta_{k+1}}(\Pi_\Omega(\mathbf{f} - \mathbf{z}_{k+1})$$

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for $k = 0, 1, \ldots, K-1$ do

$$\mathbf{z}_{k+1} = \mathcal{G}^*(\mathbf{L}_k \mathbf{R}_k^*)$$
$$\mathbf{s}_{k+1} = \mathcal{S}_{\zeta_{k+1}}(\Pi_\Omega(\mathbf{f} - \mathbf{z}_{k+1}))$$
$$\mathbf{L}_{k+1} = \mathbf{L}_k - \nabla_{\mathbf{L}} \ell(\mathbf{L}_k, \mathbf{R}_k; \mathbf{s}_{k+1}) \cdot \mathbf{P}_k$$
$$\mathbf{R}_{k+1} = \mathbf{R}_k - \nabla_{\mathbf{R}} \ell(\mathbf{L}_k, \mathbf{R}_k; \mathbf{s}_{k+1}) \cdot \mathbf{P}_k$$

end for

Dataset

I am working with a (69,69,4416) Hankel tensor with sparse entries on NYC traffic data

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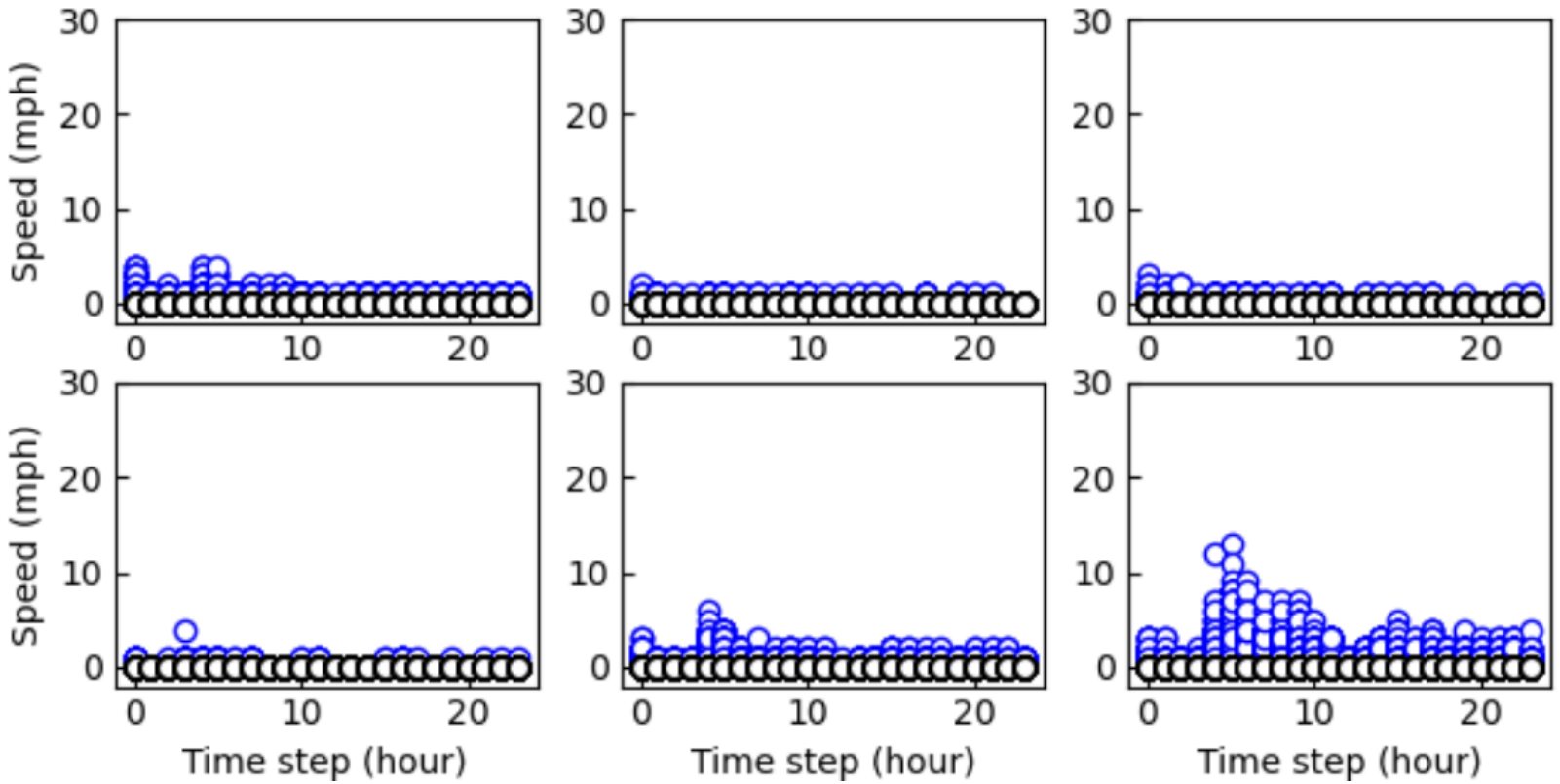


Figure: Replicated speed map from Xinyu Chen's repository

Thank You!

Questions?