Face Recognition

Lecture-14

Face Recognition

Simple Approach

- Recognize faces (mug shots) using gray levels (appearance).
- Each image is mapped to a long vector of gray levels.
- Several views of each person are collected in the database during training.
- During recognition a vector corresponding to an unknown face is compared with all vectors in the database.
- The face from database, which is closest to the unknown face is declared as a recognized face.

Problems and Solution

- Problems :
  - Dimensionality of each face vector will be very large (250,000 for a 512X512 image!)
  - Raw gray levels are sensitive to noise, and lighting conditions.
- Solution:
  - Reduce dimensionality of face space by finding principal components (eigen vectors) to span the face space
  - Only a few most significant eigen vectors can be used to represent a face, thus reducing the dimensionality

Eigen Vectors and Eigen Values

The eigen vector, x, of a matrix A is a special vector, with the following property

$$Ax = \lambda x$$

Where λ is called eigen value

To find eigen values of a matrix A first find the roots of:

$$\det(A - \lambda I) = 0$$

Then solve the following linear system for each eigen value to find corresponding eigen vector

$$(A - \lambda I)x = 0$$

Example

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 7 \end{bmatrix}$$

Eigen Values

$$\lambda_1 = 7,\ \lambda_2 = 3, \lambda_3 = -1$$

$$\mathbf{x_1} = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix},\ \mathbf{x_2} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix},\ \ \mathbf{x_3} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Eigen Vectors

Eigen Values

$$\det(A-\lambda I)=0$$

$$\det\left(\begin{bmatrix}-1 & 2 & 0\\ 0 & 3 & 4\\ 0 & 0 & 7\end{bmatrix}-\lambda\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\right)=0$$

$$\det\left(\begin{bmatrix}-1-\lambda & 2 & 0\\ 0 & 3-\lambda & 4\\ 0 & 0 & 7-\lambda\end{bmatrix}\right)=0$$

$$(-1-\lambda)((3-\lambda)(7-\lambda)-0)=0$$

$$(-1-\lambda)(3-\lambda)(7-\lambda)=0$$

$$\lambda=-1,\quad \lambda=3,\quad \lambda=7$$

Eigen Vectors

$$\lambda = -1$$

$$(A - \lambda I)x = 0$$

$$\left(\begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 0 \\ 0 & 4 & 4 \\ 0 & 0 & 8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$0 + 2x_2 + 0 = 0$$

$$0 + 4x_2 + 4x_3 = 0$$

$$0 + 0 + 8x_3 = 0$$

$$x_1 = 1, \quad x_2 = 0, \quad x_3 = 0$$

$$\mathbf{x_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Face Recognition

Collect all gray levels in a long vector *u:*

$$u = (I(1,1), \ldots, I(1,N), I(2,1), \ldots, I(2,N), \ldots, I(M,1), \ldots, I(M,N))^T$$

Collect *n* samples (views) of each of *p* persons in matrix A (*MN X pn*):

$$A = \left[u_1^1, \ldots u_n^1, u_1^2 \ldots, u_n^2, \ldots, u_1^p \ldots, u_n^p\right]$$

Form a correlation matrix L (MN X MN):

$$L = AA^T$$

Compute eigen vectors, $\phi_1, \phi_2, \phi_3, \ldots \phi n_1$, of *L*, which form a bases for whole face space

Face Recognition

Each face, u, can now be represented as a linear combination of eigen vectors

$$u = \sum_{i=1}^{n_1} a_i \phi_i$$

Eigen vectors for a symmetric matrix are orthonormal:

$$\phi_i^T . \phi_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

Face Recognition

$$u_x^T.\phi_i = \left(\sum_{i=1}^{n} a_i\ \phi_i\right)^T.\phi_i$$

$$= (a_1\phi_1^T + a_2\phi_2^T + \ldots + a_i\phi_i^T + \ldots + a_n\phi_n^T\ )\phi_i$$

$$u_x^T.\phi_i = (a_1\phi_1^T.\phi_i + a_2\phi_2^T.\phi_i + \ldots + a_i\phi_i^T.\phi_i + \ldots + a_n\phi_n^T.\phi_i)$$

$$u_x^T.\phi_i = a_i$$

Therefore: $$a_i = u_x^T.\phi_i$$

Face Recognition

L is a large matrix, computing eigen vectors of a large matrix is time consuming. Therefore compute eigen vectors of a smaller matrix, C:

$$C = A^T A$$

Let α_i be eigen vectors of C, then $A\alpha_i$ are the eigen vectors of *L*:

$$C\alpha_i = \lambda_i \alpha_i$$

$$A^T A\alpha_i = \lambda_i \alpha_i$$

$$AA^T(A\alpha_i) = \lambda_i (A\alpha_i)$$

$$L(A\alpha_i) = \lambda_i (A\alpha_i)$$

Training

- Create A matrix from training images
- Compute C matrix from A.
- Compute eigenvectors of C.
- Compute eigenvectors of L from eigenvectors of C.
- Select few most significant eigenvectors of L for face recognition.
- Compute coefficient vectors corresponding to each training image.
- For each person, coefficients will form a cluster, compute the mean of cluster.

Recognition

- Create a vector *u* for the image to be recognized.
- Compute coefficient vector for this *u*.
- Decide which person this image belongs to, based on the distance from the cluster mean for each person.

```
load faces.mat
C=A'*A;
[vectorC,valueC]=eig(C);
ss=diag(valueC);
[ss,iii]=sort(-ss);
vectorC=vectorC(:,iii);
vectorL=A*vectorC(:,1:5);
Coeff=A'*vectorL;
for i=1:30
        model(i, :)=mean(coeff((5*(i-1)+1):5*I,:));
end
while (1)
        imagename=input('Enter the filename of the image to
        Recognize(0 stop):');
        if (imagename <1)
        break;
        end;
        imageco=A(:,imagename)'*vectorL;
        disp ('');
        disp ('The coefficients for this image are:');
```

```
mess1=sprintf(‘%.2f  %.2f  %.2f  %.2f  %.2f’,
imageco(1),imageco(2),imageco(3),imageco(4),
imageco(5));
disp(mess1);
top=1;
for I=2:30
        if (norm(model(i,:)-imageco,1)<norm(model
        (top, : )-imageco,1))
        top=i
        end
end
mess1=sprintf(‘The image input was a image of person
number %d’,top);
disp(mess1);
end
```

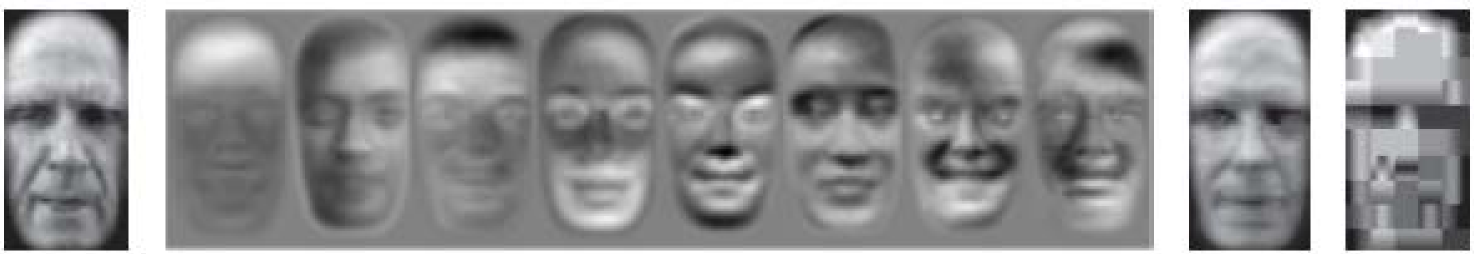
14.2 Face Recognition

Szeliski's book

Kirby and Sirovich (1990)

Face image can be compressed:

$$\tilde{x} = m + \sum_{i=0}^{M-1} a_i u_i$$



(a) (b) (c) (d)

(c) PCA Reconstruction (85 bytes)

(d) JPEG Reconstruction (530 bytes)

Scatter or Co-variance matrix: $$C = \frac{1}{N}\sum (x_j - m)(x_j - m)^T$$

Eigen Decomposition: $$C = UAU^T \frac{1}{N}\sum_{j=0}^{N-1} \lambda_i u_i u_j^T$$

Any arbitrary vector *x* can be represented: $$a_i = (x-m).u_i$$

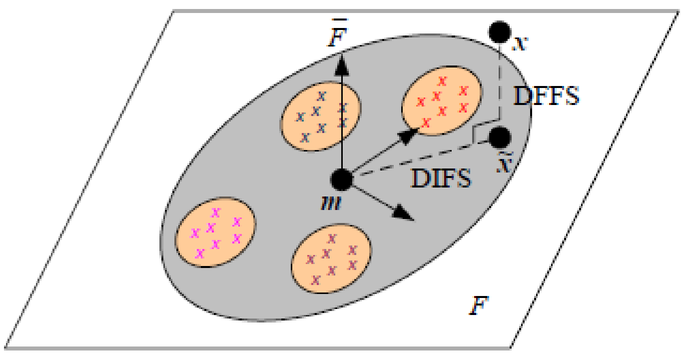
$$\tilde{x} = m + \sum_{i=0}^{M-1} a_i u_i$$

The distance of a projected face DIFS (Distance in face space)

$$DIFS = \|\tilde{x} - m\| = \sqrt{\sum_{i=0}^{M-1} a_i^2}$$

The distance between two faces

$$DIFS = \|\tilde{x} - \tilde{y}\| = \sqrt{\sum_{i=0}^{M-1} (a_i - b_i)^2}$$



We are not utilizing the eigen value information, compute Mahalonobis distance

$$DIFS' = \left\| \tilde{x} - m \right\|_{C^{-1}} = \sqrt{\sum_{i=0}^{M-1} a_i^2 \Big/ \lambda_i^2}$$

Pre-scale the eigen vectors by eigenvalues:

$$\hat{U} = U\mathrm{A}^{-1/2}$$

Euclidean $$\mathrm{d}(\mathbf{p},\mathbf{q}) = \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2}.$$

Mahalonobis $$d(\vec{x},\vec{y}) = \sqrt{(\vec{x} - \vec{y})^T S^{-1} (\vec{x} - \vec{y})}.$$

Problems in Face Recognition

- Within class variation
- Between class variation

Images Taken under Different Illuminations

Note the wide range of illumination variation, which can be more dramatic than inter-personal variations

LDA (Linear Discriminative Analysis) Fisher Faces

Slides credit:

Introduction to Pattern Analysis
Ricardo Gutierrez-Osuna
Texas A&M University

http://courses.cs.tamu.edu/rgutier/cs790_w02/l6.pdf

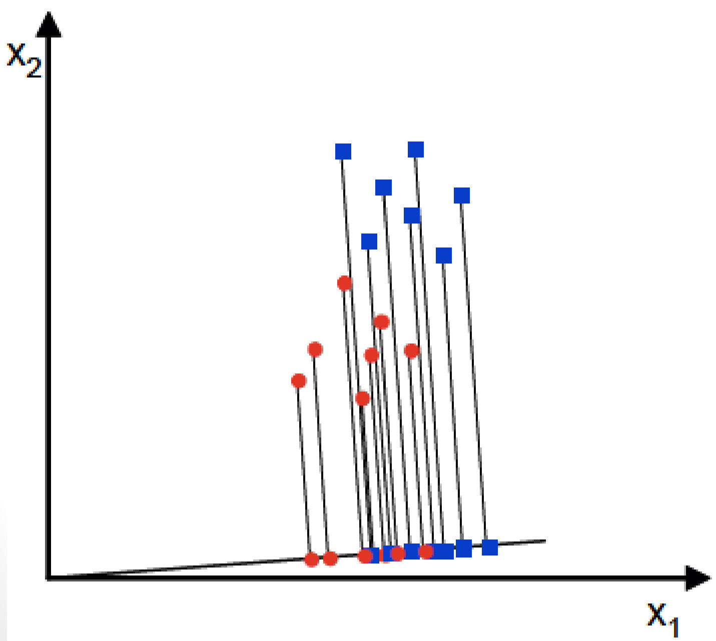
LDA

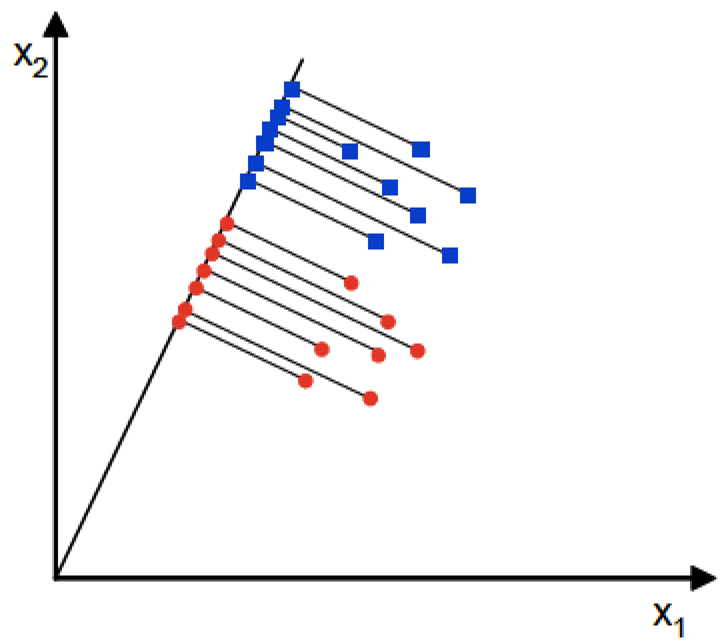
D-dimensional samples $\{x^{(1}, x^{(2}, \ldots, x^{(N}\}$

N_1 : Class - 1

N_2 : Class - 2

Project X onto a line $y = w^T x$





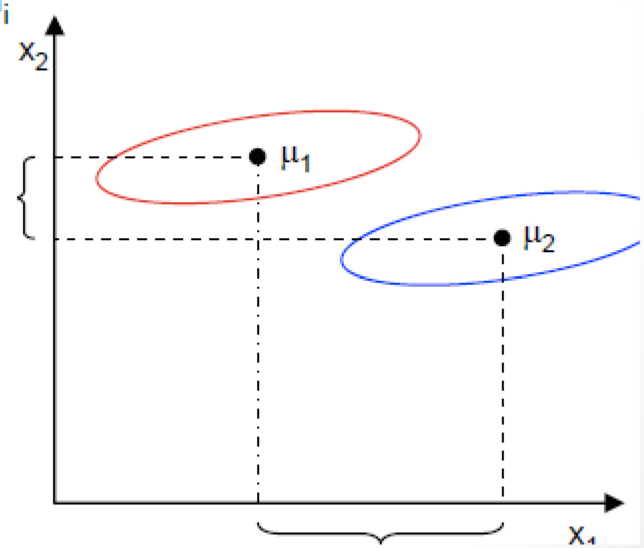
Find a measure of separation?

Distance between projected means

$$\mu_i = \frac{1}{N_i}\sum_{x\in\omega_i} x$$

$$\tilde{\mu}_i = \frac{1}{N_i}\sum_{y\in\omega_i} y = \frac{1}{N_i}\sum_{x\in\omega_i} w^T x = w^T \mu_i$$

$$J(w) = \left|\tilde{\mu}_1 - \tilde{\mu}_2\right| = \left|w^T(\mu_1 - \mu_2)\right|$$



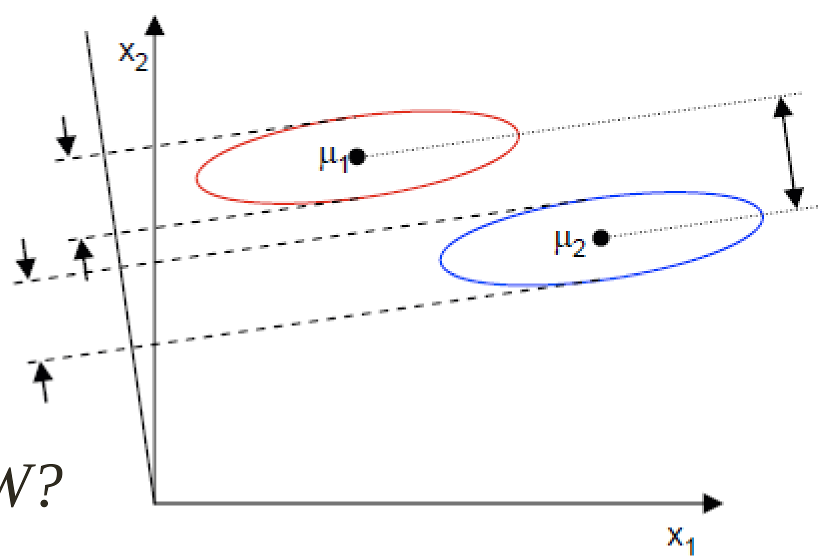
It is not good measure, since it does not consider standard deviation within a class

Maximize a function that represents the difference between the means normalized by a measure of the within-class scatter

$$\tilde{s}_i^2 = \sum_{y \in \omega_i} (y - \tilde{\mu}_i)^2$$

$\left(\tilde{s}_1^2 + \tilde{s}_2^2\right)$ is called the within-class scatter

$$J(w) = \frac{\left|\tilde{\mu}_1 - \tilde{\mu}_2\right|^2}{\tilde{s}_1^2 + \tilde{s}_2^2}$$



How to find optimum *W*?

Find Scatter Matrices

$$S_i = \sum_{x \in \omega_i} (x - \mu_i)(x - \mu_i)^T \qquad S_1 + S_2 = S_W$$

where S_W is called the **within-class scatter matrix**

Scatter Matrices for the projection

$$\tilde{s}_i^2 = \sum_{y \in \omega_i} (y - \tilde{\mu}_i)^2 = \sum_{x \in \omega_i} \left(w^T x - w^T \mu_i\right)^2 = \sum_{x \in \omega_i} w^T (x - \mu_i)(x - \mu_i)^T w = w^T S_i w$$

$$\left(\tilde{\mu}_1 - \tilde{\mu}_2\right)^2 = \left(w^T \mu_1 - w^T \mu_2\right)^2 = w^T \underbrace{(\mu_1 - \mu_2)(\mu_1 - \mu_2)^T}_{S_B} w = w^T S_B w$$

S_B is called the **between-class scatter**

$$J(w) = \frac{w^T S_B w}{w^T S_W w}$$

$$\boxed{w^* = \underset{w}{\operatorname{argmax}} \left\{ \frac{w^T S_B w}{w^T S_W w} \right\}}$$

Find Optimum W

$$\frac{d}{dW}[J(W)] = \frac{d}{dW}\left[\frac{W^T S_B W}{W^T S_W W}\right] = 0$$

$$\frac{d}{dW}[J(W)] = \frac{(W^T S_W W) 2 S_B W - (W^T S_B W) 2 S_W W}{(W^T S_W W)^2} = 0$$

$$\frac{d}{dW}[J(W)] = \frac{(W^T S_W W) 2 S_B W - (W^T S_B W) 2 S_W W}{(W^T S_W W)} = 0$$

$$\frac{d}{dW}[J(W)] = \frac{(W^T S_W W)}{(W^T S_W W)} S_B W - \frac{(W^T S_B W)}{(W^T S_W W)} S_W W = 0$$

$$\frac{d}{dW}[J(W)] = S_B W - J S_W W = 0$$

Eigen Value problem

$$\frac{d}{dW}[J(W)] = S_W^{-1} S_B W - JW = 0$$

$$S_W^{-1} S_B W = JW$$

Example

- X1=(x_1,x_2)={(4,1),(2,4),(2,3),(3,6),(4,4)}
- X2=(x_1,x_2)={(9,10),(6,8),(9,5),(8,7),(10,8)}

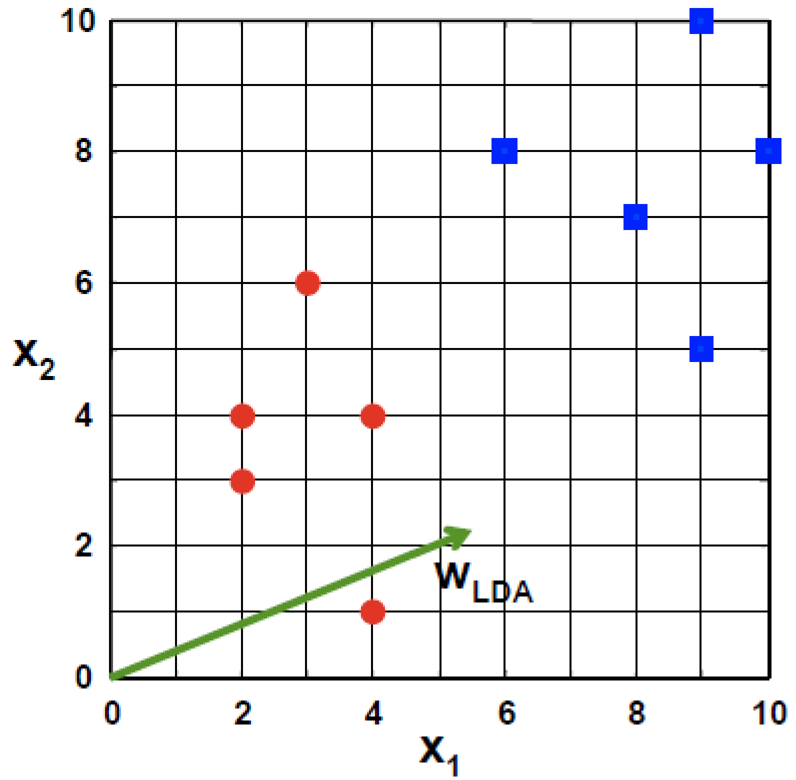
$$S_1 = \begin{bmatrix} 0.80 & -0.40 \\ -0.40 & 2.60 \end{bmatrix}; \quad S_2 = \begin{bmatrix} 1.84 & -0.04 \\ -0.04 & 2.64 \end{bmatrix}$$

$$\mu_1 = \begin{bmatrix} 3.00 & 3.60 \end{bmatrix}; \qquad \mu_2 = \begin{bmatrix} 8.40 & 7.60 \end{bmatrix}$$

$$S_B = \begin{bmatrix} 29.16 & 21.60 \\ 21.60 & 16.00 \end{bmatrix}; \quad S_W = \begin{bmatrix} 2.64 & -0.44 \\ -0.44 & 5.28 \end{bmatrix}$$

$$S_W^{-1} S_B v = \lambda v \Rightarrow \left| S_W^{-1} S_B - \lambda I \right| = 0 \Rightarrow \begin{vmatrix} 11.89 - \lambda & 8.81 \\ 5.08 & 3.76 - \lambda \end{vmatrix} = 0 \Rightarrow \lambda = 15.65$$

$$\begin{bmatrix} 11.89 & 8.81 \\ 5.08 & 3.76 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 15.65 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0.91 \\ 0.39 \end{bmatrix}$$


10
8
6
4
2
0
0
2
4
6
8
10
x_2
x_1
W_{LDA}