

CAP5415 Computer Vision Fall 2013
Homework Assignment # 1
(Due 10/1/13)

1. Given that Gaussian is given by $g(x, y) = e^{-\frac{(x^2+y^2)}{2\sigma^2}}$ derive Laplacian of Gaussian:

$$\Delta^2 G_\sigma = -\frac{1}{\sqrt{2\pi}\sigma^3} \left(2 - \frac{x^2+y^2}{\sigma^2} \right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$

2. Show that Laplacian $\Delta^2 I = I_{xx} + I_{yy}$ can also be written as $\Delta^2 = I_{\theta\theta} + I_{nn}$, where $I_{\theta\theta}$ is second derivative in direction θ and I_{nn} is second derivative in direction perpendicular to θ .
3. Show that Harris corner detector is rotation invariant.
4. Show that gradient magnitude is rotation invariant: $\sqrt{I_x^2 + I_y^2} = \sqrt{I_{x'}^2 + I_{y'}^2}$, where (x', y') are rotated coordinates of (x, y) .
5. Compare and contrast Auto Correlation matrix M in Harris detector (Slide 29, Lecture 4) and Hessian matrix H in SIFT detector (Slide 38, Lecture 5).
6. If Taylor series expansion of DOG (Difference of Gaussian) is given by

$$D(\mathbf{x}) = D + \frac{\partial D}{\partial \mathbf{x}} \mathbf{x} + \frac{1}{2} \mathbf{x}^T \frac{\partial^2 D}{\partial \mathbf{x}^2} \mathbf{x}$$

Show that the location and scale of minima/maxima is given by $\hat{\mathbf{x}} = -\frac{\partial^2 D}{\partial \mathbf{x}^2}^{-1} \frac{\partial D}{\partial \mathbf{x}}$.