CAP5415 Computer Vision Fall 2014
Homework Assignment # 1
(Due 09/30/14)

1. Given that Gaussian is given by $g(x,y) = e^{\frac{-(x^2+y^2)}{2o^2}}$ derive Laplacian of Gaussian:

$$\Delta^2 G_\sigma = -\frac{1}{\sqrt{2\pi}\sigma^3}\left(2 - \frac{x^2+y^2}{\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$

2. Show that Laplacian $\Delta^2 I = I_{xx} + I_{yy}$ can also be written as $\Delta^2 = I_{\theta\theta} + I_{nn}$, where $I_{\theta\theta}$ is second derivative in direction θ and I_{nn} is second derivative in direction perpendicular to θ .

3. Show that Harris corner detector is rotation invariant.

4. Show that gradient magnitude is rotation invariant: $\sqrt{I_x^2 + I_y^2} = \sqrt{I_{x'}^2 + I_{y'}^2}$, where (x', y') are rotated coordinates of (x, y) .

5. Compare and Contrast Autocorrelation matrix (used in Harris corner detector) and Hessian matrix (used in SIFT)

6. Drive Eigen values of Hessian matrix in terms of D_{xx} , D_{yy} , D_{xy}, D_{yx}

7. If Taylor series expansion of DOG (Difference of Gaussian) is given by

$$D(\mathbf{x}) = D + \frac{\partial D}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 D}{\partial \mathbf{x}^2}\mathbf{x}$$

   Show that the location and scale of minima/maxima is given by $\hat{\mathbf{x}} = -\frac{\partial^2 D}{\partial \mathbf{x}^2}^{-1} \frac{\partial D}{\partial \mathbf{x}}$.